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# ON ONE EFFECT OF CONTROL OF A BOUNDARY LAYER ON TRANSFER PROCESSES

T. L. PERELMAN

Heat and Mass Transfer Institute, B.S.S.R. Academy of Sciences, Minsk, B.S.S.R., U.S.S.R.

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**Аннотация**—Рассматривается «сопряжённая» [7] задача о теплообмене между пластиной и продольно обтекающей её жидкостью в случае отсасывания пограничного слоя. При этом появляется эффект недогрева жидкости у поверхности пластины [формула (16)] по сравнению со случаем обычного(неуправляемого) пограничного слоя.

## NOMENCLATURE

| | |
|---|---|
| $\theta(x, y)$, | fluid temperature; |
| $u$, | velocity component of a fluid along a plate; |
| $v$, | fluid velocity normal to a plate; |
| $U$, | incoming flow velocity; |
| $\nu$, | kinematic fluid viscosity; |
| $\chi$, | thermal diffusivity of a fluid; |
| $k_f$, | heat conduction coefficient of a fluid; |
| $k_s$, | heat conduction coefficient of a plate; |
| $\varkappa$, | ratio between heat conduction coefficients of a plate and fluid; |
| $h$, | plate thickness. |

THE SIGNIFICANCE of various methods of boundary-layer control is well known (e.g. see [1], Chapter 13 and also [2]). In particular, boundary-layer suction makes it possible to prevent separation and to decrease frictional resistance considerably.

It is obvious at the outset that suction of a boundary layer will also influence transfer processes. To estimate the influence consider the very simplest problem in transfer, namely that for a plate in a longitudinal flow with uniform boundary-layer suction (see Fig. 1).

Equations of a laminar boundary layer with suction have the general form:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{2}$$

Only the boundary condition for the velocity component along the axis changes

$$u\,|_{y=0} = 0, \tag{3}$$

$$u\,|_{y=\infty} = U, \tag{4}$$

$$v\,|_{y=0} = v_0(x) \tag{5}$$

Here ratio $\left|\frac{v_0(x)}{U}\right|$ is supposed to be small.

With uniform suction on a plate surface, condition (5) may be written

$$v_0(x) = \text{const} = -v_0 < 0. \tag{5'}$$

For sufficiently great values of the co-ordinate $x$ system (1–5) has an asymptotic solution, independent of $x$:

$$u(y) = U\,\{1 - \exp\,[-(v_0/\nu)\,y]\}, \tag{6}$$

$$v(y) = -v_0 \quad (v_0 > 0)\,* \tag{7}$$

It was shown by R. Iglisch [1] that the asymptotic profile of velocities [equations (6) and (7)] is reached at distances from the front edge of a plate given by:

$$x > \left(\frac{U}{v_0}\right)^2 \frac{\nu}{U}. \tag{8}$$

* There is a hypothesis (e.g. see [3]) that relations (6) and (7) also describe an asymptotic velocity profile in a turbulent flow. This enables one to solve the corresponding problems of transfer in a turbulent boundary layer.

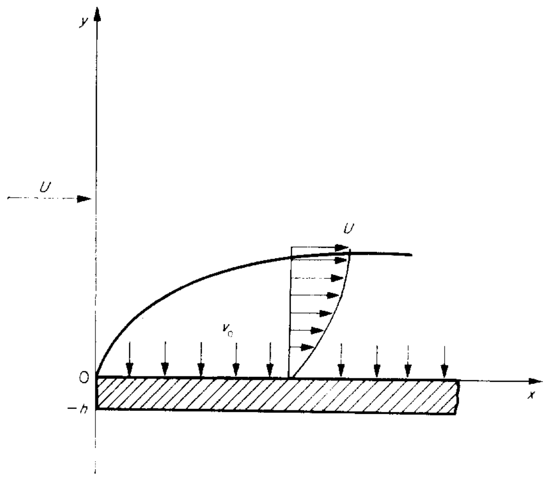


FIG. 1.

Let $\theta(x, y)$ be the fluid temperature when flowing round a plate (or the concentration of reactant for reactions occurring according to diffusion kinetics). The convective heat-transfer equation for a fluid may be written

$$U\{1 - \exp[-(v_0/\nu)\,y]\}\frac{\partial\theta}{\partial x} - v_0\frac{\partial\theta}{\partial y} = \chi\frac{\partial^2\theta}{\partial y^2}\,* \tag{9}$$

Taking temperature of incoming fluid as the temperature reference point, we have:

$$\theta\,|_{x=0} = 0 \tag{10}$$

and

$$\theta\,|_{y=\infty} = 0 \tag{11}$$

On the boundary "plate-fluid" we assume

$$\theta\,|_{y=+0} = t\,|_{y=-0}, \tag{12}$$

$$-k_f\frac{\partial\theta}{\partial y}\bigg|_{y=+0} = -k_s\frac{\partial t}{\partial y}\bigg|_{y=-0} \tag{13}$$

where $t(x, y)$ is the temperature of the plate and $k_f$ and $k_s$ are heat conduction coefficients of a fluid and plate, respectively.

Consider the influence of boundary-layer control in the case of the simple and important example of a plate $h$ thick, the lower surface of which is maintained at a constant temperature $t_0$. In [4] it was shown that the temperature $\theta(x) \equiv t(-0, y) = \theta(+0, y)$ of the upper surface, being in contact with a fluid, is expressed in terms of the normal temperature derivative on this surface

$$p(x) \equiv -\frac{\partial t}{\partial y}\bigg|_{y=-0}$$

by means of the relation

$$\theta(x) \approx t_0 - hp(x) \tag{14}$$

(to within terms, exponentially decreasing with an increase in $x/h$).

For an uncontrolled ($v_0 = 0$) laminar boundary layer the same problem has been in fact solved in [5]. For our purposes it is sufficient to recall only one physically obvious result: surface temperature $\theta(x)$ monotonically increases from 0 to $t_0$ [in turn, $p(x)$ monotonically decreases from $t_0/h$ to 0].

In our case, from simple considerations and using the maximum principle for parabolic [for fluid temperature $\theta(x, y)$] and elliptic [6] [for plate temperature $t(x, y)$] equations it may be shown that $\theta(x)$ is the limited and monotonically increasing function. Hence, it has a limit at $x \to \infty$. Taking into account the existence of a limit of the function $\theta(x)$, the asymptotic behaviour of the solution of equation (9) may be obtained.

In particular, for the outlet temperature of the surface "plate-fluid" we have:

$$\theta(\infty) = \frac{t_0}{1 + (1/\varkappa)(hv_0/\chi)} \tag{15}$$

Here $\varkappa = (k_s/k_f)$ is the ratio between the heat conduction coefficients for a plate and fluid; $\chi$ is the thermal diffusivity coefficient of the fluid; $v_0$ is the suction velocity and $h$ is the plate thickness.

* Note that in dimensionless variables of a problem $\xi = (v_0^2 x/U\chi)$ and $\eta = v_0 y/\chi$ condition (8) may be written as $\xi < Pr$ where $Pr = \nu/\chi$ is the Prandtl number (thermal or diffusion).

Thus, owing to the boundary-layer suction the fluid near a surface is underheated to a temperature $t_0$ by

$$\delta t = t_0 - \theta(\infty) = \frac{t_0}{1 + \varkappa (\chi/hv_0)} \quad (16)$$

The reasons for this effect are the sucking away of the most heated part of the fluid and also the continuous supply of a cooled fluid to the plate surface in the flow.

Note that $\delta t \to 0$ at $h \to 0$ and also at $v_0 \to 0$.

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**Abstract**—The "conjugate" problem [7] of heat transfer between a fluid and a plate in a longitudinal flow with boundary-layer suction is considered. In this connexion an effect of underheating of fluid near the plate surface [formula (16)] appears, as compared with the case of ordinary (uncontrolled) boundary layer flow.

**Résumé**—On considère le problème "conjugué" [7] du transport de chaleur entre un fluide et une plaque dans un écoulement longitudinal avec aspiration de la couche limite. Dans ce cas, un effet de surchauffe du fluide près de la surface de la plaque [formule (16)] apparaît, par comparaison avec le cas d'un écoulement de couche limite ordinaire (non contrôlé).

**Zusammenfassung**—Es wird das "konjugierte" Problem [7] des Wärmeübergangs zwischen einer Flüssigkeit und einer längsangeströmten Platte mit Grenzschichtabsaugung untersucht. Dabei ergibt sich ein Unterkühlungseffekt der Flüssigkeit nahe der Oberfläche der Platte [Gleichung (16)] im Vergleich zur gewöhnlichen (unkontrollierten) Grenzschichtströmung.